2025 **年京都大学理系問題** *1*

$\displaystyle\int_0^{\frac{\pi}{2}} \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\, d\theta$ の値を求めてください。

解説・解答

$$
\begin{aligned}
&\int_0^{\frac{\pi}{2}} \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\, d\theta \\
&= \int_0^{\frac{\pi}{2}} \sqrt{\frac{1-\left(1-2\sin^2\frac{\theta}{2}\right)}{1+\left(2\cos^2\frac{\theta}{2}-1\right)}}\, d\theta \\
&= \int_0^{\frac{\pi}{2}} \sqrt{\frac{\sin^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2}}}\, d\theta \\
&= \int_0^{\frac{\pi}{2}} \left|\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right| d\theta \\
&= \int_0^{\frac{\pi}{2}} \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} d\theta \\
&= -2\int_0^{\frac{\pi}{2}} \frac{\left(\cos\frac{\theta}{2}\right)'}{\cos\frac{\theta}{2}} d\theta \\
&= -2\left[\log\left|\cos\frac{\theta}{2}\right|\right]_0^{\frac{\pi}{2}} \\
&= -2\left(\log\frac{1}{\sqrt{2}} - \log 1\right) \\
&= \log 2
\end{aligned}
$$