2025 **年九州大学理系問題** *2*

$\displaystyle\int_0^{\frac{\pi}{4}} \frac{\tan^4\theta - \tan^2\theta - 2}{\tan^2\theta - 4} d\theta$ の値を求めてください。

解説・解答

$\tan\theta = t$ と置換します。 $\tan 0 = 0$, $\tan\dfrac{\pi}{4} = 1$

$$\frac{dt}{d\theta} = \frac{d\tan\theta}{d\theta} = \frac{1}{\cos^2\theta} = 1 + \tan^2\theta = 1 + t^2$$

$$\begin{aligned}
&\int_0^{\frac{\pi}{4}} \frac{\tan^4\theta - \tan^2\theta - 2}{\tan^2\theta - 4} d\theta \\
&= \int_0^1 \frac{t^4 - t^2 - 2}{t^2 - 4} \cdot \frac{1}{1 + t^2} dt \\
&= \int_0^1 \frac{(t^2 - 2)(t^2 + 1)}{(t^2 - 4)(t^2 + 1)} dt \\
&= \int_0^1 \frac{t^2 - 2}{t^2 - 4} dt \\
&= \int_0^1 \left(1 + \frac{2}{t^2 - 4}\right) dt \\
&= \int_0^1 \left(1 + \frac{1}{2}\Big(\frac{1}{t - 2} - \frac{1}{t + 2}\Big)\right) dt \\
&= \left[t + \frac{1}{2}\Big(\log|t - 2| - \log|t + 2|\Big)\right]_0^1 \\
&= 1 - \frac{\log 3}{2}
\end{aligned}$$