2019 年横浜市立大学医学部問題 *4*

定積分 $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{\sin^3\theta\cos\theta}\,d\theta$ を計算してください。

解説・解答

$$
\begin{aligned}
&\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{\sin^3\theta\cos\theta}\,d\theta \\
&= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cos^2\theta+\sin^2\theta}{\sin^3\theta\cos\theta}\,d\theta \\
&= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cos\theta}{\sin^3\theta}\,d\theta + \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{\sin\theta\cos\theta}\,d\theta \\
&= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cos\theta}{\sin^3\theta}\,d\theta + \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cos\theta}{\sin\theta}\cdot\frac{1}{\cos^2\theta}\,d\theta \\
&= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{(\sin\theta)'}{\sin^3\theta}\,d\theta + \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{(\tan\theta)'}{\tan\theta}\,d\theta \\
&= \Big[-\frac{1}{2\sin^2\theta}\Big]_{\frac{\pi}{6}}^{\frac{\pi}{3}} + \Big[\log|\tan\theta|\Big]_{\frac{\pi}{6}}^{\frac{\pi}{3}} \\
&= \Big(-\frac{2}{3}+2\Big) + \Big(\log\sqrt{3} - \log\frac{1}{\sqrt{3}}\Big) \\
&= \frac{4}{3} + \log 3
\end{aligned}
$$